

Formation and decay of super-heavy systems

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Received: 13 December 2001 / Revised version: 5 March 2002

Communicated by P. Schuck

Abstract. We investigate the formation and the decay of heavy systems which are above the fission barrier. By using a microscopic simulation of constrained molecular dynamics (CoMD) on Au + Au collision, we observe that composite states stay for a very long time before decaying by fission.

PACS. 24.10.Lx Monte Carlo simulations (including hadron and parton cascades and string breaking models) – 25.70.-z Low and intermediate energy heavy-ion reactions – 25.70.Jj Fusion and fusion-fission reactions – 25.70.Lm Strongly damped collisions

1 Introduction

The typical reaction mechanisms of heavy-ion collisions at lower incident energy are, depending on the energy and impact parameters, complete fusion, incomplete fusion, fusion-fission, molecular resonance, and deep-inelastic collisions. Among the huge amount of studies in this field, collisions of very heavy nuclei have been investigated mainly for the creation of a super-heavy element (SHE). SHEs are produced in two ways: one is “cold fusion” which is a complete fusion below the classical barrier, and the other is “hot fusion” which allows several neutrons to be emitted. Even though the name is “hot”, such reactions are still at very low energy near the barrier and the total mass number is very close to the aimed one. As far as the formation of SHE is concerned, the “fusion” of very heavy nuclei where the fission barrier no more exists is found to be ineffective [1,2].

Apart from the formation of SHE, the study of fission dynamics, including the spontaneous fission and the fusion-fission of heavy composites, has been one of the most important subjects. The competition of neutron emission with the fission and the fission delay have been discussed intensively. However, almost all the studies regard mass regions where the classical fission barrier exists.

Sometime ago many physicists paid attention to the low-energy collision of very heavy nuclei with regard to the spontaneous positron emission from strong electric

fields [3]. If a molecule state of, say, U and U is formed and stays for a sufficiently long time, the binding energy of an electron can exceed the electron mass and might create an electron-positron pair by a static QED process. Unfortunately no clear evidence of a static positron creation was observed below the Coulomb-energy region. In previous works it has been pointed out [4] that the nuclear reaction which causes a time delay in the separation of the two nuclei is important. Although the strong and long interaction between two nuclei increases the background component of positrons from nuclear excitations, the electron-positron from the static QED process is also expected to increase. However, the reaction mechanism of very heavy nuclei has never been discussed in fully dynamical models.

In this paper we investigate the possibility of molecule-like states of heavy nuclei and the time scales of very heavy composite system formed in fusion-fission or deep-inelastic processes. To investigate these problems theoretically we use a recently developed constrained molecular dynamics (CoMD) model [5]. The model has been proposed to include the fermionic nature of the constituent nucleons by a constraint that the phase space distribution should always satisfy the condition $f \leq 1$. Among similar molecular dynamics models, there are quantum molecular dynamics (QMD) [6], fermionic molecular dynamics (FMD) [7], and antisymmetrized molecular dynamics (AMD) [8]. QMD has been the most popular and feasible model. Unfortunately it cannot, in principle, deal with the fermionic nature of the nuclear system, although sometime a phenomenological Pauli potential is introduced for such a purpose. Therefore, the QMD model has been used mainly for higher-energy phenomena except for some studies [9,

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10]. More sophisticated models, *i.e.* FMD and AMD, deal with antisymmetrization of the wave function and have succeeded in describing nuclear reactions at medium low energy and also nuclear structures. However, due to the four-dimensional matrix element of the two-body interaction, the CPU time necessary to work out calculations for systems with total mass larger than 200 is very large for practical studies. The constrained molecular dynamics, on the other hand, can deal to a certain extent with the fermionic nature of the nuclear systems and it is still feasible for heavy systems.

In this paper we apply CoMD to the investigation of $^{197}\text{Au} + ^{197}\text{Au}$ collisions at low energies where fusion-fission or deep-inelastic process may occur. In the following section we give a brief review of the model [5].

2 Constrained molecular dynamics

The CoMD model mainly consists of two parts: classical equations of motion of the many-body system, and a stochastic process which includes the constraint due to the Pauli principle and the two-body collisions. We write the distribution function of the system as a sum of one-body distribution function neglecting the antisymmetrization:

$$f(\mathbf{r}, \mathbf{p}) = \sum_i f_i(\mathbf{r}, \mathbf{p}), \quad (1)$$

$$f_i(\mathbf{r}, \mathbf{p}) = \frac{1}{(2\pi\sigma_r\sigma_p)^3} \exp\left[-\frac{(\mathbf{r}-\mathbf{R}_i)^2}{2\sigma_r^2} - \frac{(\mathbf{p}-\mathbf{P}_i)^2}{2\sigma_p^2}\right]. \quad (2)$$

The equation of motion of \mathbf{R}_i and \mathbf{P}_i are derived using the time-dependent variational principle which gives

$$\dot{\mathbf{R}}_i = \frac{\partial H}{\partial \mathbf{P}_i}, \quad \dot{\mathbf{P}}_i = -\frac{\partial H}{\partial \mathbf{R}_i}. \quad (3)$$

In our approach the total energy H for A -particles with mass m consists of the kinetic energy and the effective interactions:

$$H = \sum_i \frac{\mathbf{P}_i^2}{2m} + A \frac{3\sigma_p^2}{2m} + V. \quad (4)$$

The second term arises from the Gaussian width in p -space. However in the following considerations we omit such a constant term.

The effective interaction V we adopt is written as

$$V = V^{\text{vol}} + V^{(3)} + V^{\text{sym}} + V^{\text{surf}} + V^{\text{Coul}}. \quad (5)$$

By defining the superimposition integral ρ_{ij} as

$$\rho_{ij} \equiv \int d^3r_i d^3r_j \rho_i(\mathbf{r}_i) \rho_j(\mathbf{r}_j) \delta(\mathbf{r}_i - \mathbf{r}_j), \quad (6)$$

$$\rho_i \equiv \int d^3p f_i(\mathbf{r}, \mathbf{p}), \quad (7)$$

the terms in eq. (5) can be written as

$$V^{\text{vol}} = \frac{t_0}{2\rho_0} \sum_{i,j \neq i} \rho_{ij}, \quad (8)$$

$$V^{(3)} = \frac{t_3}{(\mu+1)(\rho_0)^\mu} \sum_{i,j \neq i} \rho_{ij}^\mu, \quad (9)$$

$$V^{\text{sym}} = \frac{a_{\text{sym}}}{2\rho_0} \sum_{i,j \neq i} [2\delta_{\tau_i, \tau_j} - 1] \rho_{ij}, \quad (10)$$

$$V^{\text{surf}} = \frac{C_s}{2\rho_0} \sum_{i,j \neq i} \nabla_{\mathbf{R}_i}^2 (\rho_{ij}), \quad (11)$$

$$V^{\text{Coul}} = \frac{1}{2} \sum_{\substack{i,j \neq i \\ (i,j \in \text{protons})}} \frac{e^2}{|\mathbf{R}_i - \mathbf{R}_j|} \text{erf}\left(\frac{|\mathbf{R}_i - \mathbf{R}_j|}{2\sigma_r^2}\right). \quad (12)$$

In the above relations the coordinate τ_i represents the nucleon isospin degree of freedom.

We have two sets of parameters, regarding the interaction strength and the width of the distribution function, which are different mainly for the stability of the system.

The parameter set (I) used in [5], $\sigma_r = 1.3$ fm, $\sigma_p/\hbar = 0.47$ fm $^{-1}$, $t_0 = -356$ MeV, $t_3 = 303$ MeV, $\mu = 7/6$, $a_{\text{sym}} = 32$ MeV, $C_s = -0.33$ MeV fm 2 , $\rho_0 = 0.165$ fm $^{-3}$, gives a good reproduction of fragmentation data on Ca + Ca and Au + Au at 35 MeV/nucleon, and the mean radii and the binding energies in a wide range of mass.

The parameter set (II), which we introduce in this paper, $\sigma_r = 1.15$ fm, $\sigma_p/\hbar = 0.4748$ fm $^{-1}$, $t_0 = -301.1$ MeV, $t_3 = 242$ MeV, $\mu = 7/6$, $a_{\text{sym}} = 26.4$ MeV, $C_s = -0.165$ MeV fm 2 , and $\rho_0 = 0.165$ fm $^{-3}$, reasonably reproduces the fusion cross-section in Ca + Ca reactions, while set (I) overestimates such a data. Even though we have not been able to find a unique parameter set consistent with both features of fusion and fragmentation, we find some experimental confirmations of our calculations as mentioned above. In this work, since we apply the model to an energy region and to very heavy systems where the experimental data is scarce, we plan to give upper and lower estimates which will be interesting to confirm experimentally. We further strengthen our results with Boltzmann-Nordheim-Vlasov (BNV) calculations [11].

The Pauli principle is taken into account in two ways: One is the Pauli blocking of the final state of the two-body collision and the other is the constraint which brings into the system the Fermi motion in a stochastic way. The starting point of the constraint is the requirement

$$\bar{f}_i \leq 1 \quad (\text{for all } i), \quad (13)$$

$$\bar{f}_i \equiv \sum_j \delta_{\tau_i, \tau_j} \delta_{s_i, s_j} \int_{h^3} f_j(\mathbf{r}, \mathbf{p}) d^3r d^3p, \quad (14)$$

where s_i is the spin coordinate of the nucleon i . The integral is performed in an hypercube of volume h^3 in the phase space centered around the point $(\mathbf{R}_i, \mathbf{P}_i)$ with size $\sqrt{\frac{2\pi\hbar}{\sigma_r\sigma_p}}\sigma_r$ and $\sqrt{\frac{2\pi\hbar}{\sigma_r\sigma_p}}\sigma_p$ in the r and p spaces, respectively.

At each time step and for each particle i the phase space occupation \bar{f}_i is checked. If \bar{f}_i has a value greater

than unity, an ensemble K_i of nearest particles (including the particle i) is determined within the distances $3\sigma_r$ and $3\sigma_p$ in the phase space. Then we change randomly the momenta of the particles belonging to the ensemble K_i in such a way that for the newly generated sample the total momentum and the total kinetic energy is conserved (“many-body elastic scattering”). The new sample is accepted only if it reduces the phase space occupation \bar{f}_i [5].

To handle the Pauli-blocking in the collision term is straightforward from the constraint. For each NN collision we evaluate the occupation probability \bar{f}_i after the elastic scattering. If such functions for both particles are less than unity, the collision is accepted, rejected otherwise. We note that for the results discussed here and especially at the lowest energies the collision term is of little importance.

3 Collision of two ^{197}Au nuclei

To simulate the collision of two ^{197}Au nuclei, we prepare the ground state by applying the frictional cooling method together with the constraint of CoMD. The ground states we obtain have a binding energy of 7.6 MeV/nucleon and a root mean square radius of 5.76 fm with parameter set (I) and 8.4 MeV/nucleon and 5.34 fm for parameter set (II). They are rather stable for 1000 fm/c. For instance the ^{197}Au ground states with parameter sets (I) and (II) evaporate 2.75 and 3.1 nucleons during 1000 fm/c, respectively. The collision events are performed for an impact parameter b of 0 and 6 fm for an incident energy in the laboratory system of $E_{\text{lab}} = 5\text{--}35$ MeV/nucleon.

Figure 1 shows a typical event of a CoMD (I) calculation with an incident energy $E_{\text{lab}} = 10$ MeV/nucleon and an impact parameter $b = 6$ fm. The two nuclei form a quite deformed compound system, they keep such a deformation at almost 2500 fm/c and finally a fission takes place. The system does not show much rotation since the angular momentum per nucleon is not so large and the elongated shape makes the moment of inertia larger than that in the initial stage. Therefore, the reaction mechanism we are observing here may be in-between the deep-inelastic and molecular resonance.

There are many observables which distinguish the reaction mechanism. The largest fragment mass is one of such well-defined observables which can be easily measured experimentally. Figure 2 shows the time dependences of the largest cluster mass for the impact parameters $b = 0$ and 6 fm calculated by CoMD (I), CoMD (II), QMD and BNV ($b = 1$ fm and 7 fm). In CoMD calculations we see at the beginning the largest cluster mass $A_{\text{max}} = 197$ which corresponds to the projectile and to the target mass number. Within about 50 fm/c, A_{max} becomes 394 except for the incident energy $E_{\text{lab}} = 5$ MeV, which is below the barrier and the two nuclei never touch. At incident energies above the barrier, the formed system will decay into smaller fragments in different ways depending on the energy and angular momentum. At $E_{\text{lab}} \geq 30$ MeV/nucleon the largest cluster mass changes suddenly at the early stage and continuously decreases in

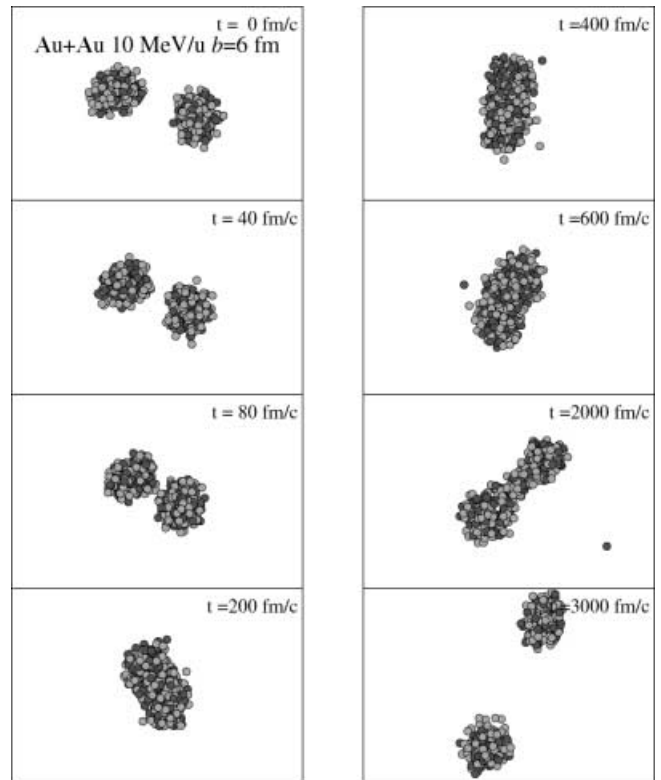


Fig. 1. Snapshot of $^{197}\text{Au} + ^{197}\text{Au}$ at $E_{\text{lab}} = 10$ MeV/nucleon, $b = 6$ fm. The time indicated in each panel is not from the contact of the two nuclei but indicates only that of the simulation.

time. This indicates multifragmentation for head-on collisions and deep-inelastic reaction for peripheral collisions followed by the emission of nucleons and small fragments. At lower incident energies ($E_{\text{lab}} \leq 20$ MeV/nucleon) there is a sudden change of the largest cluster mass at a very late time, which indicates a fission of the system. One should note that in our calculation of the Au + Au system there are almost no events where it decays only by emitting particles or light fragments, *i.e.*, pure incomplete fusion. The instability due to the Coulomb repulsion plays the major role in the decay process.

Only one event is concerned to obtain the results in fig. 2 (obviously the BNV case apart). Therefore, the fission time includes a large amount of statistical error. In fact for $E_{\text{lab}} = 10$ MeV/nucleon with $b = 0$ fm, the fission process is not observed in fig. 2(a). With different initial conditions, however, we observe the fission of the system around $t = 10^4$ fm/c for the parameter set (I). In fig. 2(c) the same quantity as fig. 2(a) and (b) is plotted for QMD calculations. These QMD calculations are based on the same code as CoMD (I) switching off the constraint procedure. The difference between the CoMD and the QMD is clear and dramatic. At low-energy collisions there are no fission processes and the system decays only by emitting nucleons and light fragments. At higher energies there are some sudden changes of the largest fragment mass even in QMD calculations. These are not due to the fission but to the partial transparency for head-on collisions or the deep-inelastic process in peripheral collisions.

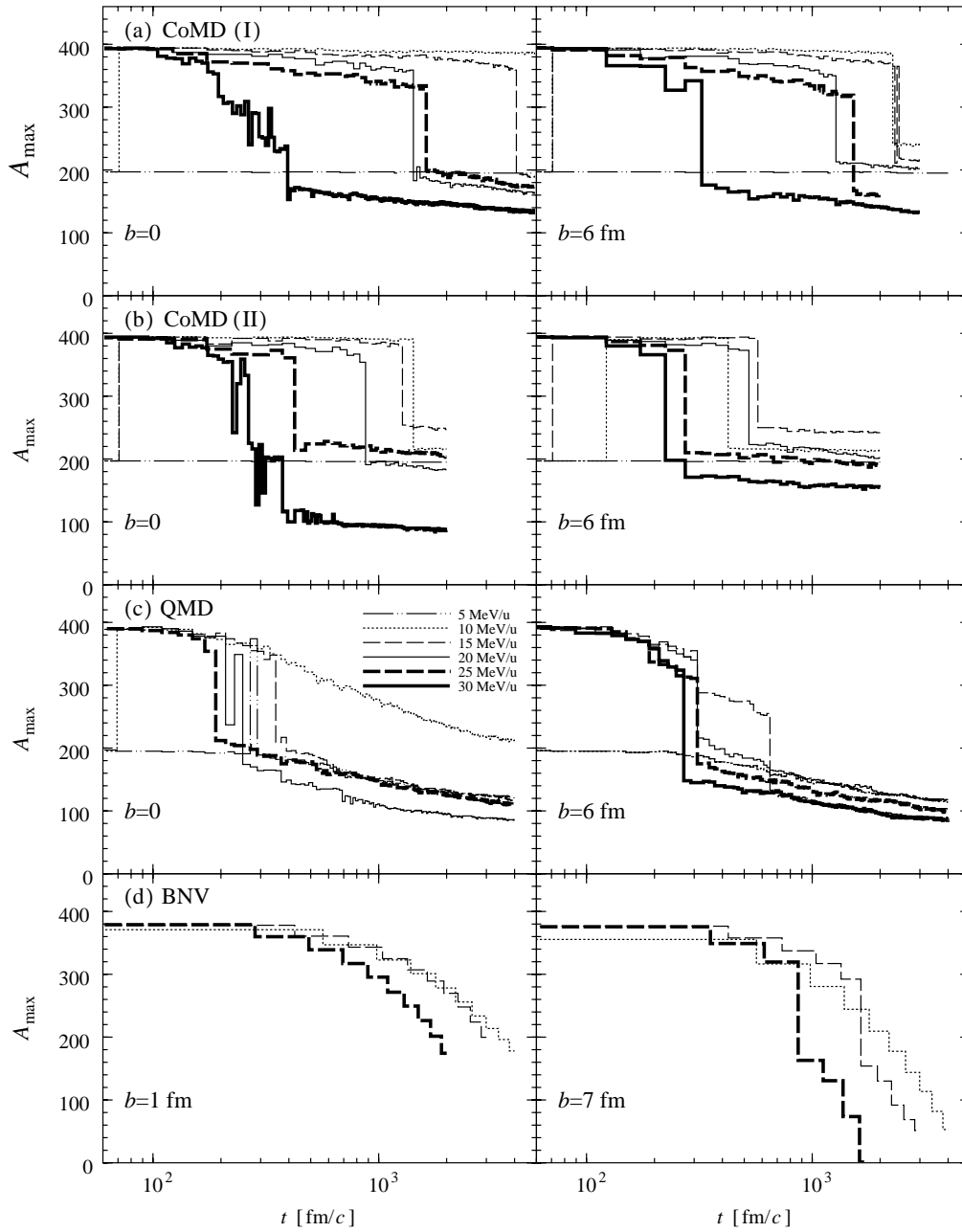


Fig. 2. The time dependence of the largest fragment mass A_{\max} . From the top (a) CoMD with parameter set (I), (b) CoMD (II), and (c) QMD. The left panels show cases of head-on collision and the right $b = 6$ fm. The lowest panels (d) refer to BNV calculations at 1 and 7 fm, respectively.

In BNV [11] calculations (fig. 2(d)) the sudden change of maximum fragment mass number is not observed for impact parameters $b = 1$ fm and $b = 7$ fm at low energy, except for $b = 7$ fm and $E_{\text{lab}} \geq 15$ MeV/nucleon. For the fission process with very small angular momentum, fluctuations and correlations are very important which are not included in the BNV calculation. Instead, the system decays via evaporation of nucleons like in the QMD case. One should note, however, that the Pauli principle is satisfied in the BNV calculation, while it is not in the QMD case. The time scale of the very large composite is still

of the same order as in CoMD (II) calculation. Although the reaction mechanism is different for CoMD and BNV, this similarity of time scales supports the validity of our CoMD calculation.

4 Lifetime of the formed composite systems

Assuming a very simple form of the time-dependent fission width $\Gamma(t) = \Gamma_f \theta(t - T_d)$, the averaged fission time T_{fiss} can be obtained from the survival probability of the compound system against two (or more) -body process

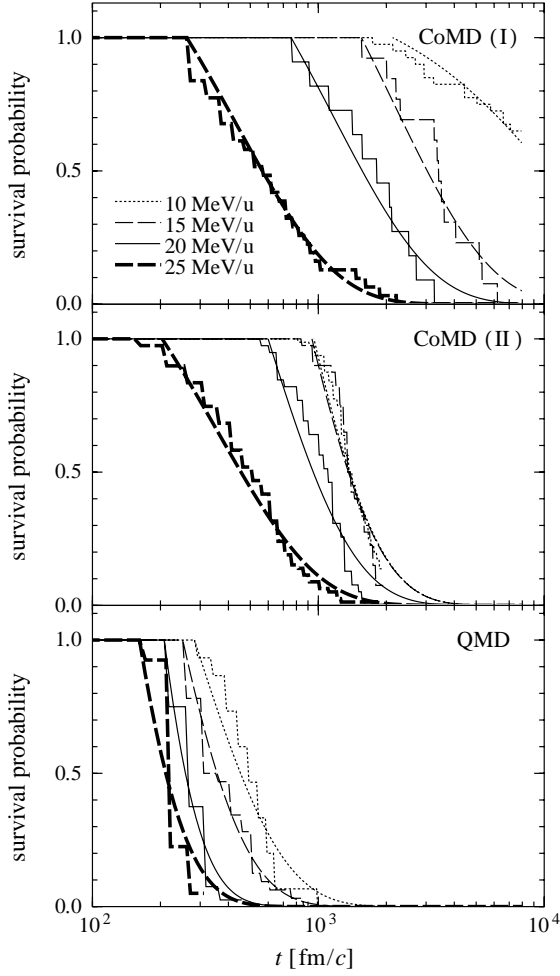


Fig. 3. The survival probability of large fragments ($A > 350$) which decay by the fission-like mode. The abscissa indicates the time after the contact of two nuclei. From the top the results are obtained for head-on collisions in CoMD with parameter sets (I), (II), and QMD. The histograms indicate results from simulations and smooth curves are the fits by eqs. (15),(16).

P_{surv} as

$$P_{\text{surv}} = \exp[-(t - T_d)\Gamma_f/\hbar], \quad (15)$$

$$T_{\text{fiss}} \equiv T_d + \hbar/\Gamma_f, \quad (16)$$

where T_d is the delay time and Γ_f is the “fission width” after the delay time. The probability $P_{\text{surv}}(t)$ is obtained directly from the simulation. Here t denotes the time after the contact of two nuclei. The fitting applies well for fission-like processes. Figure 3 shows the survival probability of a large fragment with $A > 350$ which decays by a fission-like mode or by emitting large fragments ($A > 30$). The histograms are directly obtained in the simulation and the curves are the fits by eqs. (15),(16). From the top, results of CoMD with parameter set (I), (II) and QMD are listed for the impact parameter $b = 0$ and for several incident energies $E_{\text{lab}} = 10\text{--}25$ MeV/nucleon. For all the calculations the fitting works well, particularly the effect of the delay time. The assumption of constant fission width after the delay time, on the other hand, is not

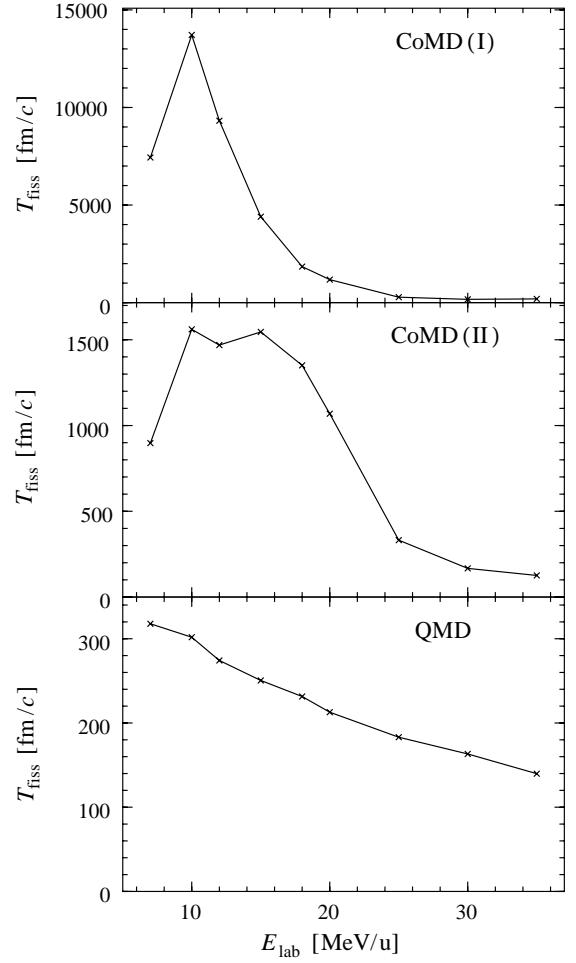


Fig. 4. The fission lifetime obtained from eqs. (15),(16).

completely supported because of poor statistics and still existing dynamical effects. One should note that the fitting by eqs. (15),(16) is just to extract the “fission” time of the super-heavy composite. Especially, the time scale of the QMD results is obviously different from that of the fission process.

The extracted fission times T_{fiss} are plotted in fig. 4. The fission times shown for parameter set (I) might be too long for such heavy system. To make more quantitative discussions, we should improve the effective interaction. Using the parameter set (II), we obtain smaller values of the fission time. We can consider the obtained values as upper and lower limits of the fission times in our CoMD model. However, the experimental data will finally support one or the other result which, we stress, are both qualitatively similar and somehow surprising. For both parameter sets the longest lifetime of very heavy composite is found at $E_{\text{lab}} = 10$ MeV/nucleon.

For lower incident energies (just above the Coulomb barrier) the system might not form a fully thermalized single composite but might be quasi-separated in the phase space, which makes the system split easily. For higher energies, the fully thermalized system needs some fluctuations to reparate even though there is no classical barrier

for fission. Therefore, the fission time gets shorter with the increase of the incident energy.

In QMD calculation what is in marked contrast to the CoMD calculation is that the “fission time” has no maximum energy and shows a monotonic decrease. This is due to the lack of the Pauli principle which prevents the two nuclei from overlapping at low energies just above the Coulomb barrier.

For peripheral collision ($b = 6$ fm), the lifetime of the very heavy composite is shorter than the head-on collisions. But the incident-energy dependence is very similar to the $b = 0$ fm cases. Though the mechanism is much more dynamical, eqs. (15),(16) fit well again.

Nevertheless, the super-heavy composite system formed in head-on collisions of Au + Au may survive for a rather long time of 10^3 – 10^4 fm/c. We also note that long-lived strongly deformed (see fig. 1) systems have been observed in binary dissipative collisions between lighter system ($A_{\text{total}} \simeq 60$) in the same scaled energy regime with respect to the Coulomb barrier. The long lifetimes have been estimated through the comparison of the incident-energy-averaged angular distributions and/or the excitation functions with the results of the partially overlapped molecular level model (POMLM)[12]. Such approaches could be extended to the present case.

Another interesting aspect of the long-lived very heavy system is, as mentioned before, the spontaneous positron-electron production from the strong electric field as a static QED process. The total charge of the Au + Au system may be still smaller than the necessary charge ($Z \sim 170$) for this process. However, the nuclear reaction of, e.g., the U + U system, should be qualitatively the same as what we observe in the Au + Au system. Although the background positrons should be larger, one can get the longest lifetime of the strong electric field (stronger than the case of Rutherford or molecular trajectory) around $E_{\text{lab}} = 10$ MeV/nucleon at some impact parameter and the production of positrons from the static QED process should be largest around that energy.

5 Asymmetric fission of the composite systems

As mentioned above, the production of SHE is one of the most important subject in heavy-ion collisions. Besides cold and hot fusion, mass transfer in collisions of very heavy nuclei was tried before. One could produce, e.g., up to Fm ($Z = 100$) in the U + U system, or Md ($Z = 101$) in the U + Cm system, by such a mechanism [1,2]. The incident energy, however, was very close to the Coulomb barrier and the reaction was rather gentle with the transfer of ~ 20 nucleons. In our CoMD calculation for $E_{\text{lab}} \geq 7$ MeV/nucleon, the reaction mechanism is more violent which results in the transfer of much more nucleons even though the mass loss from the system is also large. In fig. 5 plotted is the mass asymmetry $(A_1 - A_2)/(A_1 + A_2)$ of the fission process in CoMD calculation. Here A_1 and A_2 are the largest and the second largest fragment mass when the fission occurs. The mass-asymmetry increases

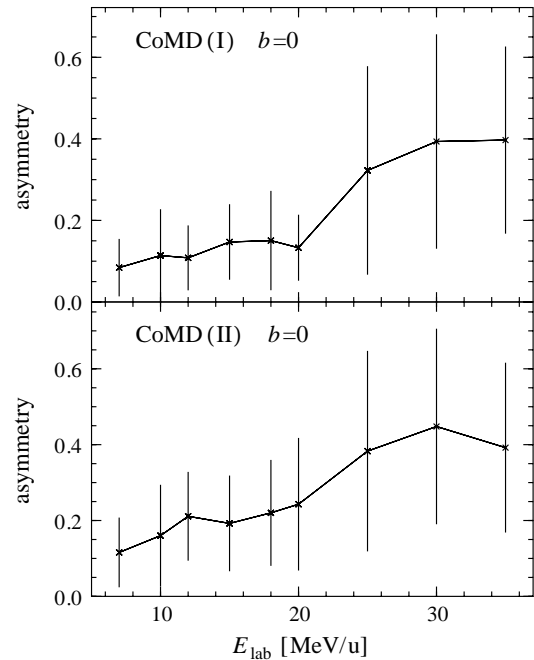


Fig. 5. The mass asymmetry of the fission fragments. Error bars indicate the statistical standard deviation.

with the incident energy. At $E_{\text{lab}} = 7$ MeV/nucleon, the asymmetry amounts to about 0.1 and at 10 MeV/nucleon almost 0.2 as average. If we simply assume no proton loss and asymmetry of 0.2, the largest fragment charge will be 112 for the U + U system. Of course we should consider the thermal mass loss and the subsequent fission due to the excitation of fragments. However, our proposed reactions around 10 MeV/nucleon should be taken into account for the SHE production. The new 4π detectors can accumulate lots of statistics plus they can make coincidence studies to see if the fragments come from fission.

6 Summary

In summary, we have discussed the formation and decay of super-heavy composites in the Au + Au collisions. The CoMD calculations which take into account the fermionic nature of the nucleon many-body system can describe well the low-energy dynamics including fusion, fission, deep-inelastic, emission of nucleons and small fragments, and multifragmentation. Although there are still some ambiguities on the effective interaction, the lifetime of super-heavy composites is found to be rather long up to 10^3 – 10^4 fm/c. Some experimental explorations such as the detection of e^+e^- formation at around 10 MeV/nucleon and the measurement of the energy-averaged angular distribution and/or the excitation function for binary processes are encouraged.

One of the authors (T.M.) thanks INFN-LNS for the warm hospitality during his stay and Dr. A. Iwamoto, Dr. H. Ikezoe, and Dr. S. Mitsuoka for fruitful discussions. A.B. thanks Prof. J. Natowitz for enlightening discussion on the super-heavy system discussed here.

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